Chapter 5

The Inner Magnetosphere

5.1 Trapped Particles

The motion of trapped particles in the inner magnetosphere is a combination of gyro motion, bounce motion, and gradient and curvature drifts. In particular in the inner magnetosphere ($r < 6R_E$) this is an appropriate model because the magnetic field is dominated by the dipole component and relative magnetic field changes are small.

5.1.1 Bounce Motion

Knowledge of the bounce motion in the magnetic dipole field is needed for a quantitative evaluation of the drift (gradient and curvature) motion. This motion is determined by the mirror points and the equatorial pitch angle: In the equatorial plane the magnetic field strength is $B_{eq} = B_E/L^3$. With the magnetic field at the mirror points (3.14) the equatorial pitch angle is

$$\sin^2 \alpha_{eq} = \frac{B_{eq}}{B_m} = \frac{\cos^6 \lambda_m}{\left(1 + 3\sin^2 \lambda_m\right)^{1/2}} \tag{5.1}$$

Note that the equatorial pitch angle depends on the the latitude of where a particle is reflected.

Bounce Period:

The bounce period is the time it for a particle to move from one mirror point to the other and back.

$$\tau_b = 4\int_0^{\lambda_m} \frac{ds}{v_{||}} = 4\int_0^{\lambda_m} \frac{ds}{d\lambda}\frac{d\lambda}{v_{||}} \tag{5.2}$$

as derived in appendix 1 the bounce time is

$$\tau_b = 4\frac{r_{eq}}{v}\int_0^{\lambda_m} \cos\lambda\sqrt{1 + 3\sin^2\lambda}\left[1 - \sin^2\alpha_{eq}\frac{\sqrt{1 + 3\sin^2\lambda}}{\cos^6\lambda_m}\right]^{-1/2} d\lambda \tag{5.3}$$

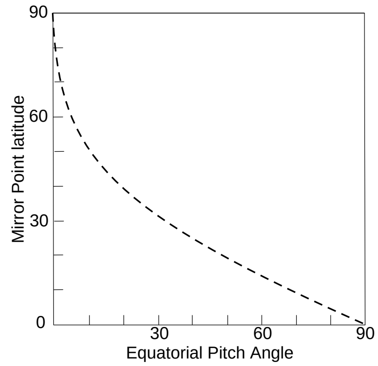

Figure 5.1: Mirror point latitude as function of equatorial pitch angle.

Numerically the integral is $\Gamma_a \approx 1.3 - 0.56 \sin \alpha_{eq}$ and the bounce period becomes

$$\tau_b \approx \frac{LR_E}{(W/m)^{1/2}} \left(3.7 - 1.6 \sin \alpha_{eq}\right) \tag{5.4}$$

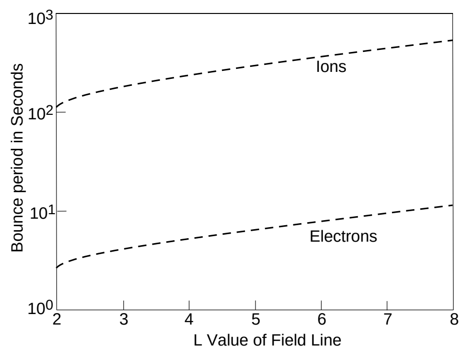

Figure 5.2: Bounce period as a function of L value for 1 keV particles and $\alpha_{eq} = 30$.

Evaluating the bounce period for 1 keV particle and equatorial pitch angle of 30° yields a few to 10 seconds for electrons and 2 to 6 minutes for the bounce period. Note that the second invariant may be violated for ions by geomagnetic pulsations of few minute periods which are quite common.

Loss cone:

Not all particles participate in the bounce motion. Particles with a mirror point at or below ionospheric heights will be absorbed in the atmosphere through collisions with the neutral atmosphere. Assuming a mirror point at $1R_E$ yields an equatorial loss cone

$$\sin^2 \alpha_l = \frac{B_{eq}}{B_E} = \frac{\cos^6 \lambda_E}{\left(1 + 3 \sin^2 \lambda_E\right)^{1/2}} \tag{5.5}$$

All particles with $\alpha < \alpha_l$ and all particles with $\alpha > 180° - \alpha_l$ will be absorbed in the Earth's atmosphere. The field line equation gives the latitude at the Earth's surface for a particular L value as $\cos^2 \lambda_E = 1/L$ such that the loss cone becomes

$$\sin^2 \alpha_l = \left(4L^6 - 3L^5\right)^{-1/2} \tag{5.6}$$

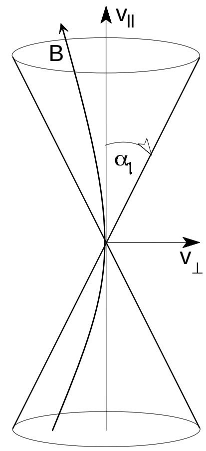

Figure 5.3: Illustration of the loss cone.

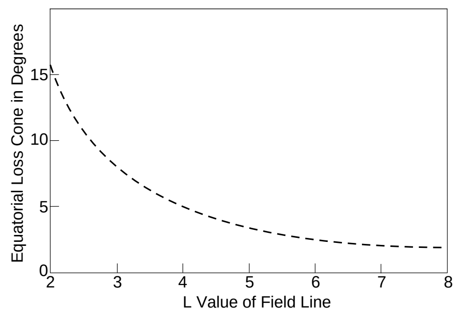

Figure 5.4: Equatorial loss cone as a function of L value.

5.1.2 Particle Drift Motion

During gyro and bounce motion a particle will also undergo gradient and curvature drift motion. The instantaneous angular drift velocity is $v_d/r\cos\lambda$. With $dt = ds/v_{||}$ the drift angle over a bounce motion is

$$\Delta\psi = 4\int_0^{\lambda_m} \frac{v_d}{r\cos\lambda}\frac{ds}{v_{||}} \tag{5.7}$$

The angular velocity of the average drift motion is

$$\langle\omega_d\rangle = \frac{\Delta\psi}{2\pi\tau_b}$$

With $W = mv^2/2$ and the integrals I_1 and I_2 derived in appendix 2 the average drift becomes

$$\langle\omega_d\rangle = \frac{3WL}{2\pi q B_E R_E^2}\frac{I_2(\alpha_{eq})}{I_1(\alpha_{eq})} \tag{5.8}$$

The ratio of the integrals can be approximated by (according to Parks, Physics of Space Plasmas)

$$\frac{I_2(\alpha)}{I_1(\alpha)} \approx 0.35 + 0.15\sin\alpha_{eq}$$

Which yields a drift period of

$$\langle \tau_d \rangle \approx \frac{2\pi q B_E R_E^2}{3WL} \left(0.35 + 0.15 \sin \alpha_{eq}\right)^{-1} \tag{5.9}$$

and an average equatorial drift velocity of

$$\langle v_d \rangle \approx \frac{3WL^2}{qB_E R_E} \left(0.35 + 0.15 \sin \alpha_{eq}\right) \tag{5.10}$$

Comparison with particles with $\alpha_{eq} = 90°$:

$$\begin{aligned} \mathbf{v}_\nabla &= \frac{m v_\perp^2}{2qB^3} \left(\mathbf{B} \times \nabla B\right) \\ &= \frac{3WL^2}{qB_E R_E} \end{aligned} \tag{5.11}$$

shows the same dependence but a factor of 2 difference. I suppose the ratio I_2/I_1 requires a correction by a factor of 2, i.e., $I_2/I_1 = 0.7 + 0.3 \sin \alpha_{eq}$.

Exercise: Compute the average drift velocity and an expression for the integral I_2 following the same procedure but use the radius of curvature to evaluate the local drift.

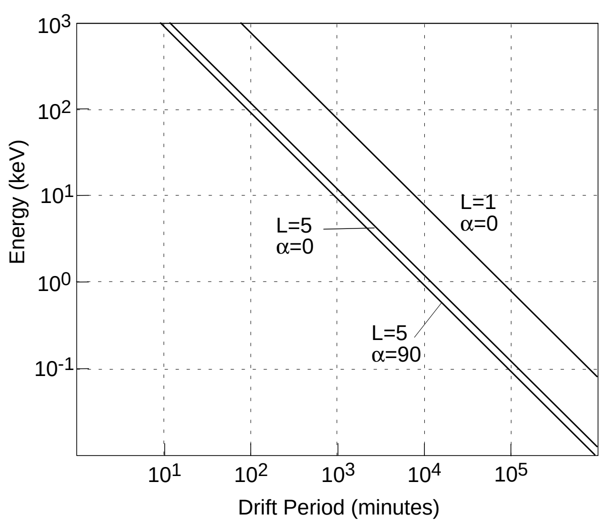

Figure 5.5: Drift period in the Earth's dipole field.

The expression demonstrates a relatively weak dependence on the pitch angle α_{eq}. The drift period is

$$\langle \tau_d \rangle \approx \begin{cases} 43.8/LW & \alpha_{eq} = 90° \\ 62.7/LW & \alpha_{eq} = 0° \end{cases} \tag{5.12}$$

Notes:

- 1 keV particle has drift periods of 100 to several hundred hours. On these time scale the third adiabatic invariant is likely violated.
- The drift velocity scales with L^2 => drift periods are shorter for larger L values.
- Energy dependence leads to a separation of particles with different energies (if injected at the same location).
- Pitch angle dependence also generates some separation of particles with different pitch angles.
- Typical ring current particles (10 keV) do not complete a full revolution.
- Only MeV particles orbit fast enough to complete closed orbits
- Electrons and ions drift in opposite directions thereby generating the so-called ring current.

Electric drift

In addition to the gradient curvature drift there are two other particle drifts which need to be considered for the particle motion in the inner magnetosphere:

Corotation of the magnetic field: This drift can be expressed as $\mathbf{v}_E = \omega_E r \mathbf{e}_\varphi$ with the angular velocity of the Earth's rotation ω_E and the eastward unit vector $\mathbf{e}_\varphi$. The corresponding electric field $\mathbf{E} = -\mathbf{v}_E \times \mathbf{B}$ in the equatorial plane can be expressed in terms of the corotation potential $\Phi_{cor} = -\omega_E B_E R_E^3 / r$. This potential implies that particles are rotating with the Earth.

Convection: For simplicity we assume a constant electric field E_0 across the magnetosphere. While this is not entirely realistic it provides insight into the basic effects of convection in the equatorial plane. Using the angle φ measured from the sun-Earth line in the Eastward direction on the dayside, the corresponding potential is $\Phi_{conv} = -E_0 r \sin\varphi$. Finally we will confine the motion to the equatorial plane, i.e., consider only particles with $90°$ pitch angle. The corresponding gradient B drift potential is $\Phi_\nabla = \mu B_E R_E^3 / \left(q r^3\right)$. The resulting total drift velocity is

$$\mathbf{v}_D = \frac{1}{B^2} \mathbf{B} \times \nabla \Phi_{tot}$$

with $\Phi_{tot} = \Phi_{conv} + \Phi_{cor} + \Phi_\nabla$. The gradient drift is small compared to the corotation for sufficiently low particle energies (Figure 5.5). Thus the net particle motion is given by the sum of the corotation potential and the convective potential $\Phi_{pp} = \Phi_{conv} + \Phi_{cor}$. The resulting drift paths (contours of constant Φ) are sketched in Figure 5.6.

Close to the earth the drift paths are circular while they become almost straight lines at large distances. The direction of corotation and convection are opposite to each other on the duskside which generates the bulge in the closed (trapped) drift region. The location of zero velocity is $r_{pp}^2 = \omega_E B_E R_E^3 / E_0$. Thus the region of trapped particles shrinks in size for large values of the convection electric field. The region inside the separatrix represents the plasma sphere, a region with relatively cold plasma confined to a few R_E around the Earth. The separatrix or boundary of the trapped low energy population is called the plasma pause. Observations show that the plasma sphere indeed shrinks in size during times of magnetic

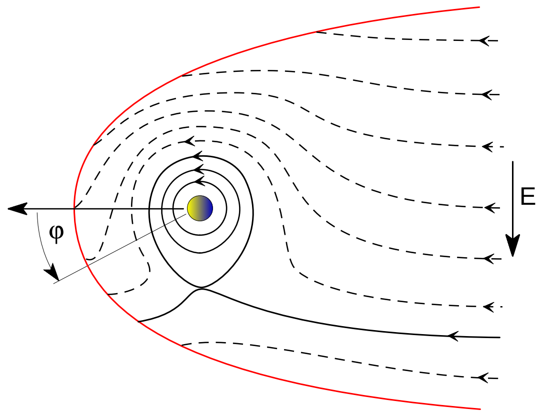

Figure 5.6: Low energy particle drifts in the equatorial magnetosphere.

activity and satellite observations demonstrate the disappearance of much of the plasma sphere during strong substorms.

The motion of energetic particle can be constructed in a similar way. For these particles one can neglect the corotation potential in favor of the gradient drift potential. The resulting motion is given by $\Phi_{rc} = \Phi_{conv} + \Phi_{\nabla}$ and the drift paths are sketched in Figure 5.7.

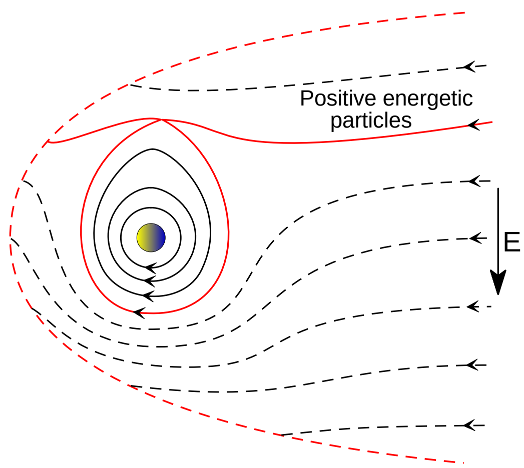

Figure 5.7: Energetic ion drift paths in the equatorial plane.

Energetic particles are trapped closer to the Earth inside the separatrix (in red). The radial distance for the zero flow condition is $r_{rc} = 3\left(\mu B_E R_E^3 / |q|\ E_0\right)^{1/4}$. As in the case of low energy particles the trapped region shrinks for strong convection which dominates in the outer magnetosphere. However, different from the case of low energy particles, the size of the trapping region increases with particle energy. Note that the drift is charge dependent such that the corresponding configuration for energetic electrons is mirrored across the noon midnight meridian. The separatrix between trapped and non-trapped particles is called the Alfvén layer.

5.1.3 Sources and Sinks of Ring Current Particles

The drift due to a cross tail electric fields indicates that the plasma sheet is a source for the ring current particles.

Adiabatic heating:

During the particle drift motion particles encounter stronger magnetic fields. Since the magnetic moment is conserved during this motion particles become adiabatically heated. If a particle starts at an L shell L_0 this heating is

$$\frac{W_\perp}{W_{\perp 0}} = \left(\frac{L_0}{L}\right)^3 \tag{5.13}$$

where $W_{\perp 0}$ is the initial particle energy. This heating is substantial. A particle starting from 8 R_E doubles it energy by drifting to 6 R_E and increases it energy by a factor of 20 when it drifts to $L = 3$.

Similarly a particle can be heated in the parallel direction through conservation of the second adiabatic invariant. In approximate solution for this heating is

$$\frac{W_{||}}{W_{||0}} = \left(\frac{L_0}{L}\right)^\kappa \tag{5.14}$$

with $\kappa = 2$ for $\alpha_{eq} = 0°$ and $\kappa = 2.5$ for $\alpha_{eq} \approx 90°$. Adiabatic heating is a main contributor to the more energetic 10-100 keV ring current particles.

During the drift the different perpendicular and parallel heating generates a change in anisotropy. With $\kappa = 2.25$ this anisotropy change is

$$\frac{A_W}{A_{W0}} = \left(\frac{L_0}{L}\right)^{0.75} \tag{5.15}$$

There are other mechanisms which can lead to particle heating such as transient exposure to large amplitude waves and electric fields. However, these as well as the question of source regions are still a matter of ongoing research.

Loss Processes:

Similar to the source processes, loss mechanisms are still researched. A major loss process is charge exchange with neutrals. Plasma densities are relatively small of the order of a few hundred particles per cm^3. Coulomb collisions are too infrequent to account for losses. However, ions can undergo charge exchange collisions with neutrals after which a low energy ion and a high energy neutral are formed. The neutral may fall back into the atmosphere or escape the magnetosphere. The low energy ion carries hardly any current and will eventually be absorbed in the atmosphere. Other loss processes include interaction with waves which can scatter particles in velocity space (pitch angle scattering) or interchange motion.

5.2 Ring Current

The drift current for particles with an equatorial pitch angle of $\alpha_{eq} = 90°$, energy W, and density n on a given L shell is

$$j_d = \frac{3L^2 nW}{2B_E R_E}$$

which represents a current in the westward direction.

5.2.1 Magnetic disturbance:

We can estimate the total current $I_L$by noting that $I_L dl = j_d dV$ with the circumference element dl and the volume element dV. Assuming that I_L is not φ dependent and with $\int dl = 2\pi L R_E$ the total current is

$$I_L = \frac{3U_L L}{2\pi B_E R_E^2}$$

with the total energy $U_L = \int nW dV$. The magnetic perturbation at the Earth's center can be computed from Biot-Savart's law

$$\delta B_d = -\frac{\mu_0 I_L}{2LR_E} = -\frac{\mu_0}{4\pi}\frac{3U_L}{B_E R_E^3}$$

The minus sign implies a magnetic field opposite to the Earth's dipole magnetic field. This magnetic perturbation does not depend on the distance of the drifting plasma such that one can replace U_L with the total energy of all drifting particles U_R. The resulting disturbance field is

$$\Delta B_d = -\frac{\mu_0}{4\pi}\frac{3U_R}{B_E R_E^3}$$

To determine the total magnetic magnetic disturbance one must include diamagnetic effects. The gyro motion of a charge particle represents a current loop which generates a magnetic perturbation. Assuming a particle gyrating in the equatorial plane with a pitch angle of $\alpha_{eq} = 90°$ the resulting perturbation at the Earth's center is

$$\delta B_\mu = -\frac{\mu_0}{4\pi}\frac{\mu}{L^3 R_E^3}$$

The magnetic moment at a distance LR_E is $\mu = mv^2/2B_{eqL} = mv^2 L^3/2B_E$ such that the magnetic disturbance from a single particle is

$$\delta B_\mu = -\frac{\mu_0}{4\pi}\frac{W}{B_E R_E^3}$$

Integration over all particles yields

$$\Delta B_d = \frac{\mu_0}{4\pi}\frac{U_R}{B_E R_E^3}$$

such that the combined disturbance from the particle drifts and the diamagnetic effect becomes

$$\Delta B = -\frac{\mu_0}{2\pi}\frac{U_R}{B_E R_E^3}.$$

Thus the magnetic disturbance is a measure of the total energy of the ring current particles. Note, however, that other magnetospheric currents such as the magnetopause, mantle, tail and ionospheric currents also contribute to the magnetic disturbances.

The total magnetic energy of a dipole field is

$$U_{mag} = \frac{4\pi}{3\mu_0} B_E^2 R_E^3$$

This can be used to express the magnetic disturbance field as

$$\frac{\Delta B}{B} = -\frac{2}{3}\frac{U_R}{U_{mag}}$$

5.2.2 Magnetic Storms

Caused by large solar flares or coronal mass ejections the terrestrial magnetosphere can become strongly perturbed for long periods of time (days). This disturbance caused by a strong increase in ring current energy is most easily measured at equatorial latitudes where local ionospheric currents do not cause the large perturbations which are present at high latitudes. The average magnetic perturbation near the equator establishes the *Dst index*.

A geomagnetic storm is defined by a strong depression of the equatorial magnetic field by several 100 nT. This corresponds often to disturbances of more than 1 % of the dipole magnetic field.

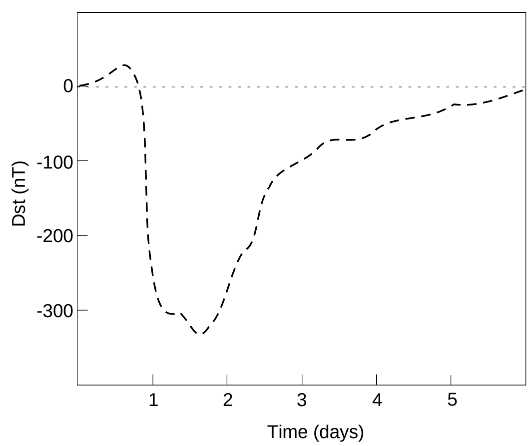

Figure 5.8: Equatorial magnetic field depression (Dst) during a magnetic storm.

Storms have two phases, the first being the strong increase increase of the ring current particles. This usually requires long duration (many hours) of southward IMF conditions. During this the average electric fields in the magnetosphere are much larger than usual. They lead to the increase of ring current particles and contribute to the heating of the plasma. After a day or two the IMF conditions return to normal and the ring current decays on time scales of a few days. Keeping in mind that other currents also contribute to the magnetic disturbance one can assume that at least half of the depression during storms is caused by the intensification of the ring current. Thus the associated energy can be estimated to

$$\Delta U_R \geq \frac{\pi}{\mu_0} B_E R_E^3\, Dst$$

This expression yields an increase in ring current energy of $2 \cdot 10^{13}$ J per 1 nT depression of the magnetic field. Assuming that the current is concentrated at $L = 5$ a current of $4 \cdot 10^4$ A produces a 1 nT depression. During a big magnetic storm the ring current energy increases by more than $5 \cdot 10^{15} J$ and the total current can assume more than 10^7 A. With an average energy of 10 keV per particle more than $3 \cdot 10^{30}$ particles are injected into the ring current at such times.

All of the consideration in this and in the previous chapter are based on single particle dynamics. This is justified as long as the changes in the magnetic field which are caused by the currents are small because the solution of the particle motion assumes a fixed magnetic field. Within the magnetosphere this is best justified in the inner magnetosphere where the magnetic field is usually much larger than the magnetic perturbation caused by the local current. Thus it is possible to calculate typical properties and explain magnetic perturbations and particle dynamics in terms of the test particle approach. However, it should be kept in mind that there are many plasma processes also in the inner magnetosphere such as waves, shocks, and particle acceleration parallel to the magnetic field which require the collective behavior and cannot be addressed within this test particle approach.