31. Switch-off shock - Properties

Switch-off shocks are an important structure in simple models for magnetic reconnection.

(a) Compute the compression ratio for the limiting cases of the shock angle $\theta = 0$ and $\theta = \pi/2$.

(b) The dimensionless reconnection rate r is typically of $O(0.1)$ and is approximately $r = \cos\theta$ (i.e., θ is close to $\pi/2$). Expand the solution for the compression ratio around $\theta = \pi/2$ to include the lowest order correction for r and show that the compression ratio is approximately

$$X = 1 + \frac{1}{\gamma\beta + \gamma - 1}\left(1 - \frac{(\gamma\beta - 1)}{\gamma(\beta + 1)}r^2\right)$$

(c) Show that the amplification of the thermal pressure is given by

$$\frac{p_d}{p_u} = X\left[1 + \frac{\gamma - 1}{\gamma\beta}\left(1 - \frac{r^2}{X^2}\right)\right]$$

and compute the ratio of the entropy function $s = p/\rho^\gamma$.

(d) What are the compression, the pressure, and the entropy ratio in the limit of $\beta = 0$ and $\beta \gg 1$? Can you get large ratios for the entropy function in the case of high plasma β?

32. Switch-off shock - Simulation

Modify the the initial condition 1 to introduce a separate equilibrium density, pressure and magnetic field such that the initial configuration becomes $B_y = B_0 \tanh x$, $\rho = \rho_0 + B_0^2/\cosh^2 x$ and use $p = p_0 + B_0^2/\cosh^2 x$ and choose $B_0 = 1$, $\rho_0 = 1$, and $p_0 = 1$ as a reference. Run this case for with $B_x = 0$ and $B_x = 0.1$ (note, you have to run this probably for several 100 simulation times to see the evolution clearly.

(a) The two step waves that propagate in the positive and negative x direction are slow switch-off shock. Examine the properties and compare these with the results for the plasma compression and pressure ratio.

(b) Sketch the situation. What transformation velocity is needed for a transformation into a frame in which the electric field is 0?

(c) What changes are needed to simulate low and a high β cases. Apply these changes and run one low and one high β case. Are the results consistent with the prediction for the low and high beta limits for the compression and pressure ratio.

(d) run the low beta case for `intu` $= 0$ and 2. How do the results differ?

33. Project:

Continue the work on your project. Provide another progress report for this week.

Please turn in the solutions to the homework on Friday, 4/18/08