1. Energy conservation in MHD:

Derive the conservative form of the total energy density equation in MHD (section 3.4.3 in the manuscript).

2. Harris Sheet Equilibrium:

Consider the pressure function $p(A_z) = p_0 \exp(-2A_z/A_c)$ and $B_z = 0$. Show that a one-dimensional $\partial/\partial y = \partial/\partial z = 0$ solution of Grad-Shafranov equation (for two-dimensional equilibria, equation 3.36)

$$\Delta A_z = -\mu_0 \frac{dp(A_z)}{dA_z}$$

has the form $A_z(x) = A_c \ln\cosh(x/L)$ and that the corresponding magnetic field is the Harris sheet field.

3. Dipole Field:

Determine the vector potential component A_ϕ for the dipole field and derive the equations for the magnetic field lines in spherical coordinates. The dipole field is given by

$$\begin{aligned} B_r &= -\frac{\mu_0 M_D}{2\pi}\frac{\cos\theta}{r^3} \\ B_\theta &= -\frac{\mu_0 M_D}{4\pi}\frac{\sin\theta}{r^3} \end{aligned}$$

Please turn in the solutions to the homework on Friday, 11/17/2006