1. Amdahl's law.

Consider a vector processor speed of 100 times the scalar speed ($V = 100S$). Using Amdahl's law, plot the speed-up in execution time (on a log scale) as a function of the fraction P of the program that is vectorized. Assuming a total of 10^6operations, how many of these can remain scalar to achieve speed-ups of 10 and 90?

2. Parallel Processing.

(a) Following the method in class derive the number of surface gridpoints N_s (for a typical processor subdomain) for a two-dimensional simulation with $N = n^2$gridpoints and $N_p$parallel processors for the one-dimensional and the two-dimensional domain decomposition (processors arranged in a square geometry with $\sqrt{N_p}$ processors along each side). Assuming perfect parallelization ($P = 1$, $F = 1$) show that the parallel execution time for this situation (analogous to equation 1.3 of the manuscript) is given by

$$\tau_p = 2d\frac{N^{1/2}}{N_p^\alpha}\tau_c + \frac{N}{N_p}\tau_s$$

with $\alpha = \left\{0, \frac{1}{2}\right\}$ and $d = \{1, 2\}$ for the 1D and 2D domain decompositions. Assume a communication time of $\tau_c = 10\tau_s = 10^{-6}$ ($\tau_s = 1/S$, i.e., the single processor execution time for a 1 floating point operation), and $N = 10^6$.

b) Plot the ratio of the total communication time and the total computation time as a function of N_p for 1 to 1000 processors (ignore the fact that the processor number along a row should be integer). Use a (base10) logarithmic scale from 10^{-2} to 10 for the vertical axis. Discuss shortly your results for the 2 domain decompositions.

Determine the number of processors for each of the two values of α for which half of the total time is spent on communication.

c) Plot and discuss the total execution times τ_p for the two cases. How do these properties for change for $N = 10^{10}$.

3. Physical equations.

a) Derive the pressure equation (2.21) from the energy (2.20), momentum (2.19), and continuity equation (2.18).

b) Re-write the pressure equation as an equation for temperature using the ideal gas law $p = nk_BT$ and an isotropic pressure $\Pi = p\underline{\underline{1}}$. What other assumption do you need to obtain thetemperature diffusion equation as presented by (2.27)?

Please turn in the solutions to the homework on Monday 2/5/2006.