20. The mass operator M_x for quadratic finite elements is determined by the elements $m_{ij} = \int \phi_i \phi_j dx$. On even nodes these elements are given by $\phi_{A,B} = 0.5\xi(\xi \pm 1)$ and for odd nodes they are $\phi = (1 - \xi^2)$. The transformation between x and ξ is given by

$$\begin{aligned} \xi_A &= 2\frac{x - 0.5(x_{j-2} + x_j)}{x_j - x_{j-2}} \\ \xi_B &= 2\frac{x - 0.5(x_{j+2} + x_j)}{x_{j+2} - x_j} \end{aligned}$$

Determine the mass operator for an evenly spaced grid. (Note: It is sufficient to compute three elements of this operator if you consider the symmetry of the problem.)

21. Viscous flow in a rectangular duct is governed by

$$\left(\frac{b}{a}\right)^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + 1 = 0$$

subject to the boundary conditions $w = 0$ at $x = \pm 1$, $y = \pm 1$. The exact solution for this problem is given by

$$w = \left(\frac{8}{\pi^2}\right)^2 \sum_{i=1,3,5..}^{L} \sum_{j=1,3,5..}^{L} \left[\frac{(-1)^{(i+j)/2-1}}{ij(ib/a)^2 + j^2} \cos(0.5i\pi x)\cos(0.5j\pi y)\right]$$

with sufficiently large L. Obtain approximate solutions using (a) the Galerkin, (b) the subdomain, and (c) the collocation method. As an approximate solution, choose

$$w = \sum_{j=1}^{N} a_j (1 - x^2)^j (1 - y^2)^j$$

which is suggested by an appropriate form of the limiting one-dimensional velocity profiles ($b/a = 0$ and ∞). Obtain approximate solutions for $N = 1$ and 2. Compare these with the exact solution $L = 21$. Comment your

22. Select a class project. Try to formulate goals for the project and become familiar with the methodology. Provide a brief report (not more than three pages) on this progress with your project. Particularly list also any problems you may have encountered. This report as any future reports should demonstrate that you have indeed thought about the problem and made first progress (or not if documented by a corresponding discussion of the problems).

Please turn in the solutions to the homework on Friday, 4/04/2005